Conflict-free Collaborative Set Sharing for Distributed Systems

Masato Takeichi

takeichi@acm.org

November 26, 2021

Abstract. Collaborative Data Sharing is widely noticed to be essential for distributed systems. Among several proposed strategies, *conflict-free* techniques are considered useful for serverless concurrent systems.
They aim at making shared data be consistent between peers in such a way that their local data do not become equal at once, but they arrive at the same data eventually when no updates occur in any peer.
Although the *Conflict-free Replicated Data Type* (CRDT) approach could be used in data sharing as well, it puts restrictions on available operations so as to concurrent updates never cause conflicts. Even for sets, popular operations such as insertion and deletion are not freely used, for example.
We propose a novel scheme for *Conflict-free Collaborative Set Sharing* that allows both insertion and deletion operations. It will provide a new synchronization method for data sharing and gives a fresh insight into designing conflict-free replicated data types. We might consider that this becomes a substitute for CRDTs.

1 Introduction

In the server-client system, clients can easily share data on the server. This is a traditional style of data sharing. However, in distributed systems where each site, or peer, has its own exclusive property of the contents and the policy, data sharing between peers for collaborative work raises several substantial problems.

We have been discussing "What should be shared" in collaborative data sharing, but not so much talking about "How should be shared".

Concerning the "what", a seminal work on *Collaborative Data Sharing* [9, 10] brought several issues upon specification of data to be shared. An approach based on the view-updating technique with *Bidirectional Transformation* [4, 2, 5, 7, 6] has been proved promissing. Among others and aside from data transformation, we observe different approaches on the "how": The *Dejima* 1.0 implementation [8, 1] uses the PostgreSQL synchronization mechanism with *strong consistency* between distributed data. And the *BCDS Agent* [15] implementation applies a "happy-go-lucky" technique for *eventual consistency* based on the property of *Bidirectional Function Composition* [14], but we face some difficulty to foresee the final result because of its global nature.

In collaborative data sharing, each peer has its local data and it provides some of them to other peers and receives some data from peers for reflecting updates on these shared data.

From the other side of data sharing, peers may start with the common data as shared and locate its replicas as their own local data. And then each peer performs local operations on the replica and does any way to synchronize it with other peers. In this course, how to share data between peers is same as how to synchronize distributed replicas to be the same. Thus, they are almost equivalent except original intentions. And our problem to be solved is how to synchronize distributed replicas in serverless distributed systems.

We have various kinds of *Conflict-free Replicated Data Types* (CRDTs) [12]. The CRDT approach restricts available operations acted on replicated data; the *Grow-Only-Set* (G-Set) CRDT allows only the insertion operation on the set data, for example.

We can observe another kind of collaborative systems in daily life, i.e., Real-time Collaborative Document Editors. Most systems are implemented with the use of the *Operational Transformation* (OT) technique [3, 13]. The OT for text editing deals with a string as the replica and the operations on the replica should be aware of the position in the string. The synchronization process is known to be rather complex and error-prone. And most of these systems use the server for synchronization. So, it is not appropriate for us to add emphasis on OT for data sharing in general, while we come across its basic idea in our scheme.

In this paper, we will explore a novel scheme for set data sharing between distributed replicas. The set data is the base of various data types and it is ubiquetous in that it spreads over many applications and it can be extended in many ways.

Our *Conflict-free Collaborative Set Sharing* (CCSS) allows general set operations, i.e., insertion and deletion of an element, and avoids any conflicts between concurrent operations to realizes eventual consistency. This is the most distinguished feature of our CCSS compared with CRDTs.

2 Whereabouts of Conflicts in Distributed Systems

Consider a data sharing example of distributed systems: Peers P and Q have their local data D_P and D_Q to be appropriately *synchronized*. That is, P and Q have replicas D_P and D_Q respectively as instances of the same set. P and Q update D_P and D_Q respectively whether or not the network connection is alive, and they try to synchronize them during the connection is alive.

Each peer inserts element x into its local data (written as $\cup\{x\}$) and deletes element from it (written as $\setminus\{x\}$), and sends the operations thus performed to the partner peer. This is the *client* function of the peer. The peer as the *server* receives remote operations from the partner peer and puts them on the local data so that it becomes same as that of the partner peer.

What happens in the events?

1. Start with $D_P = D_Q = \{1, 2\}$.

2. Network connection fails.
3. P does $\cup\{3\}$ and then $\setminus\{3\}$.
4. Q does $\setminus\{2\}$ and then $\cup\{3\}$.
5. Connection is restored.

How are D_P and D_Q synchronized? And what is the result after Step 5? Is $D_P = \{1, 2\}$ or $\{1, 3\}$? Is $D_Q = \{1, 3\}$ or $\{1, 2, 3\}$?

It would be reasonable to answer this question with expectation as "It should be $D_P = D_Q = \{1, 3\}$."

This small example may remind us of conflict resolutions for data sharing in distributed systems.

2.1 How CRDT Solves the Problem

The Conflict-free Replicated Data Type (CRDT) approach [12] follows "When in Rome do as the Romans do （郷に入っては郷に従う）". That is, the local data D of type CRDT is defined by restricting operations on D so that it never becomes inconsistent with others upon updates.

There are two types of CRDT approaches: operation-based CRDTs and state-based CRDTs. It is known that the above two are equivalent. Since we will propose a scheme based on operations, we give here an overview of operation-based CRDTs.

Operation-based (op-based) CRDTs [11] places data type operations into messages, which are sent to all replicas in order. The peers apply received operations to their replica so that they all arrive at the same state, even if they receive concurrent messages in different orders.

The *Grow-Only-Set* (G-Set) allows only the insertion operation $\cup\{x\}$[1].

The local data of P and Q were synchronized before connection failure and they have been modified with local operations $\langle\cup\{p_1\}, \cup\{p_2\}, \cdots, \cup\{p_m\}\rangle$ and $\langle\cup\{q_1\}, \cup\{q_2\}, \cdots, \cup\{q_n\}\rangle$ respectively during the connection failure period.

After the connection is established again, P and Q send their local operations to each other for synchronization. Then, P as the server applies the remote operations received from Q to the current local data $D_P \cup \{p_1\} \cup \{p_2\} \cdots \cup \{p_m\}$ to obtain $D_P \cup \{p_1\} \cup \{p_2\} \cdots \cup \{p_m\} \cup \{q_1\} \cup \{q_2\} \cdots \cup \{q_n\}$. Similarly Q has now the new local data $D_Q \cup \{q_1\} \cup \{q_2\} \cdots \cup \{q_n\} \cup \{p_1\} \cup \{p_2\} \cdots \cup \{p_m\}$. It is easy to show that these are same provided that $D_P = D_Q$ before the connection failure. This is because the commutative property $D \cup \{x\} \cup \{y\} = D \cup \{y\} \cup \{x\}$ holds for any x and y.

As illustrated above in the G-Set CRDT, CRDTs solve consistency problems by only allowing *monotonic* updating operations; any operation must make the structure larger.

[1] The operation is applied from the left of the operand D, i.e., $D \cup \{x\}$ represents the set data obtained by inserting x into D. Thus, all the operations in this paper are postfixed after the operand data.

To define a Set CRDT with insertion and deletion, we have to do something for deletion since deletion breaks monotonicity. A simple idea called *Two-Phase-Set* (2P-Set) CRDT is to use a pair (A, R) of two G-Sets for the local data D: A for inserted (added) elements and R for deleted (removed) elements. Deletion never actually removes elements, but does mark them as deleted and keep them in the second G-Set R of "tombstones". When an element x in the local data $D = (A, R)$ is "deleted", A remains unchanged, i.e., A does not become $A \setminus \{x\}$, while R grows to $R \cup \{x\}$. Thus, the deletion operation is also monotonic and D does not shrink.

Then, how can we answer the question: "What is the actual elements of the local data D?" In this CRDT, we should answer that "It is $A \setminus R$". That is, all the elements of A not included in R, are the actual existent elements.

We cannot effectively insert elements again into the 2P-Set after they have been deleted sometime before, since $A \setminus R$ always excludes elements ever deleted from A.

As an oft-cited example of a shopping basket in an online shop:
- The G-Set cannot be used, for we cannot remove items once added to the cart.
- The 2P-Set cannot be practical, for we might finally want to add items that were once added to the cart but deleted sometime before.

For the example above in this Section, the 2P-Set CRDT may cause difficulties. If we use 2P-Sets for D_P and D_Q, starting from $D_P = D_Q = \{1, 2\}$, after P does $\cup\{3\}$ and then $\setminus\{3\}$ and Q does $\setminus\{2\}$ and then $\cup\{3\}$, we see that 3 is never included actually in D_P and D_Q since P's operation $\setminus\{3\}$ rejects actual 3 whether it precedes or follows Q's operation $\cup\{3\}$.

From these observations, we note that we should develop another schema for sharing set data with useful operations available, insertion and deletion, under our usual interpretation.

3 Conflict-free Collaborative Set Sharing

The basic idea behind our *Conflict-free Collaborative Set Sharing* (CCSS) is to take particular note on the fact that our object data is *mutable* and operations on that data should be closely related to the current state (value) of the data.

Usual mathematical definitions of operations $\odot x$ on set D refers to the operator $\odot$ that maps as $(D, x) \mapsto D'$ independently of D with x. But we define here $\odot x$ itself by making full use of the relationship between D and x like $x \in D$, for example.

Consider another small example using usual set operators:
- Assume that P and Q have local data D_P and D_Q which have been synchronized as $D_P = D_Q = \{1, 2\}$.
- And then, P wants to "insert 2" into D_P by a postfix operation $\cup\{2\}$ and Q tries to "delete 2" from D_Q by operation $\setminus\{2\}$.
- When both operations finish, D_P remains unchanged while D_Q has been changed into $\{1\}$.

How can we make D_P and D_Q be synchronized, i.e., are made the same? We have no clue for synchronization unless something is given other than the current data. If we are given the operations $\cup\{2\}$ on D_P and $\setminus\{2\}$ on D_Q, we can use them for understanding the intenstions.

Since $D_P = D_Q$ when P and Q began to perform concurrent operations, we should observe that $\cup\{2\}$ and $\setminus\{2\}$ cause a *conflict* of effective update: Which should be taken for obtaining consistent D_P and D_Q?

To answer this question, consider the reason why P wanted to "insert 2" into D_P. Supposedly, P wanted to share 2 with Q by means of and at the time of the next synchronization. As it were, what happened if P had checked before taking the operation whether "$2 \in D_P$"? If P noticed that 2 was already in D_P, P had nothing to do for that purpose because the intended state had already been there.

If it were, no conflict occurs!

This is what leads us to the idea of using *effectful* set operations for our Conflict-free Collaborative Set Sharing.

3.1 Effectful Set Operations

We assume that our mutable data D is a set of elements x of any type, and operations $\oplus x$ and $\ominus x$ on D change the value D into $D \oplus x$ and $D \ominus x$, respectively. Of course, we can read this mathematically as D is mapped to $D \odot x$, i.e., $(D, x) \mapsto D \odot x$ by $\odot x$. We are using a generic operator symbol $\odot$ for representing $\oplus$ or $\ominus$.

We call insertion operation $\oplus x$ *effectful* if it gives $D \oplus x \neq D$. That is, the effectful operation $\oplus x$ is defined and it can be applied to D only if $x \notin D$. Similarly the effectful $\ominus x$ is defined and it can be applied to D only if $x \in D$, and then $D \ominus x \neq D$.

In short, the effectful operations always change D when they are defined and applied to D, while usual set operation $D \cup \{x\}$ and $D \setminus \{x\}$ do not always.

For convenience, we introduce a postfix identity operation "!" which does not change the value, i.e., $D! = D$ for any D. In fact the operation "!" is not effectful according to the above intuitive meaning, but we will use this for the "do nothing" operation.

Properties of effectful set operations For the operations $\oplus x$ and $\ominus x$,

- $D \oplus x \oplus x$ and $D \ominus x \ominus x$ *never appear* since the second occurences of $\oplus x$ and $\ominus x$ after the first same operation are *not defined.* Thus, the validity of using the effectful operations depends on the context.
- $D \ominus x \oplus x = D$ and $D \oplus x \ominus x$ hold as long as they are valid, i.e, $x \in D$ in the first case, and $x \notin D$ in the second case. We can read this as effectful $\oplus x$ and $\ominus x$ cancel each other.
- For $x \neq y$, hold

$$D \oplus x \oplus y = D \oplus y \oplus x$$

$$D \oplus x \ominus y = D \ominus y \oplus x$$

$$D \ominus x \ominus y = D \ominus y \ominus x$$

This means that $(\oplus x, \oplus y)$, $(\oplus x, \ominus y)$ and $(\ominus x, \ominus x)$ are *commutative.*

3.2 Normalization of Operation Sequence

We write a sequence of operations as $\langle \odot x_1, \odot x_2, \cdots \rangle$ where each $\odot$ may not be the same; so it represents $\langle \oplus x_1, \oplus x_2, \cdots \rangle$, $\langle \oplus x_1, \ominus x_2, \cdots \rangle$, ..., etc.

From the above canceling rule $D \oplus x \ominus x$ and the commutativity rule of $\oplus x$ and $\ominus x$, if $\oplus x$ and $\ominus x$ appear in this order with no $\odot x$ in-between,

$$D \odot \cdots \oplus x \cdots \ominus x \cdots = D \odot \cdots ! \cdots ! \cdots$$

holds. In fact, the occurences of $\oplus x$ and $\ominus x$ may be removed from the sequence. But we use the identity operation "!" to fill the positions to keep the length of the sequence.

Same for the case that $\ominus x$ and $\oplus x$ appear in this order.

Thus, we can *normalize* the operation sequence into one that does not contain cancel-able pairs of $\oplus x$ and $\ominus x$, and they are replaced with "!". This does not cause any effect on the data.

And from now on, we assume that the operation sequence has been normalized.

As a consequence of normilization, no duplicate appears in the normalized sequence $\langle \odot x_1, \odot x_2, \cdots \rangle$. This is because that if ever there were pairs of operations satisfying $x_i = x_j$, i.e., the same operation appear at different positions, they must be $(\oplus x_i, \ominus x_j)$ or $(\ominus x_i, \oplus x_j)$, since neither $(\oplus x_i, \oplus x_j)$ nor $(\ominus x_i, \ominus x_j)$ appears from the definition of effectful operations. But this contradicts the assumption that the operation sequence has been normalized.

3.3 Synchronization of Effectful Operations

Assume that P and Q share data D by locating its replicas D_P and D_Q as their local data. And they independently and concurrently perform local operations $\odot p$ and $\odot q$ respectively on their replicas.

Also assume that P as the client has performed local operations $\odot p_1$, $\odot p_2$, $\cdots$, $\odot p_m$ on local data D_P to get the current data $D_P \odot p_1 \odot p_2 \odot \cdots \odot p_m$. Similarly Q has got $D_Q \odot q_1 \odot q_2 \odot \cdots \odot q_n$ by operations $\odot q_1$, $\odot q_2$, $\cdots$, $\odot q_n$.

Then, what should be done for synchronizing P's replica and Q's replica so that they contain the same data?

P as the server receives remote operations $\odot q_1$, $\odot q_2$, $\cdots$, $\odot q_n$ from Q to make the local data reflect these remote operations. A simple-minded way to do this might be applying remote operations to the current data as

$$D_P \odot p_1 \odot p_2 \odot \cdots \odot p_m \odot q_1 \odot q_2 \odot \cdots \odot q_n.$$

However, this sometimes fails because the effectful operation $\odot q_j$ $(j = 1, 2, \cdots, n)$ is not always valid in this expression. Recall that the effectful $\oplus x$ can be applied to D only if $x \notin D$ and $\ominus x$ can be applied to D only if $x \in D$, but $\odot q_j$ may violate these side conditions when it is applied to P's replica, while $\odot q_j$ is valid in Q's replica.

Therefore we need to transform each remote operation q_j into an effectful q'_j that reflects the effect of $\odot q_j$ on the current data.

Confluence of updates by synchronization Given normalized operation sequences $\boldsymbol{ps} = \langle \odot p_1, \odot p_2, \cdots, \odot p_m \rangle$ and $\boldsymbol{qs} = \langle \odot q_1, \odot q_2, \cdots, \odot q_n \rangle$, P as the server calculates $\boldsymbol{qs'} = \langle \odot q'_1, \odot q'_2, \cdots, \odot q'_n \rangle$ according to the following rule: For each $j = 1, 2, \cdots, n$, if operation q_j appears in $\boldsymbol{ps}$, then set $q'_j :=!$ else set $q'_j := q_j$.

Using this $\boldsymbol{qs'}$, the current data in P is updated into

$$D_P \odot p_1 \odot p_2 \odot \cdots \odot p_m \odot q'_1 \odot q'_2 \odot \cdots \odot q'_n. \tag{1}$$

This expression does not violate the validity of effectful operations.

It is almost the same in Q. We can apply the same algorithm by exchanging $\boldsymbol{ps}$ and $\boldsymbol{qs}$ and calculating $\boldsymbol{ps'}$. The current data in Q is now updated into

$$D_Q \odot q_1 \odot q_2 \odot \cdots \odot q_n \odot p'_1 \odot p'_2 \odot \cdots \odot p'_m. \tag{2}$$

Apart from procedural operations, we can do another transformation in P as if its local data were not yet updated by $\boldsymbol{ps}$. The above algorithm for P can be rewritten for obtaining $\boldsymbol{ps'}$ as: For each $j = 1, 2, \cdots, n$, if operation q_j appears at some position, say k in $\boldsymbol{ps}$, then set $p'_k :=!$ else set $p'_k := p_k$.

As long as the final value is concerned, we have

$$D_P \odot p'_1 \odot p'_2 \odot \cdots \odot p'_m \odot q_1 \odot q_2 \odot \cdots \odot q_n. \tag{3}$$

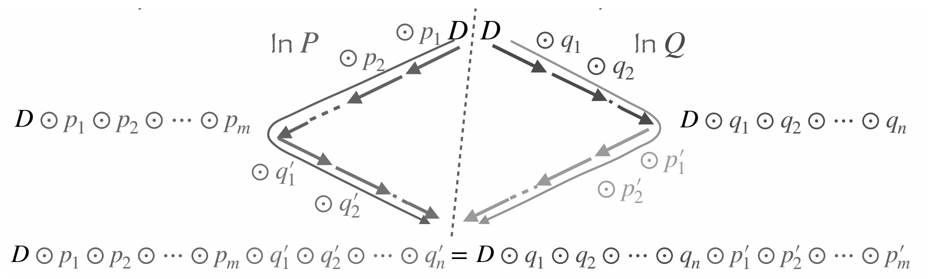
by first applying operations $\boldsymbol{ps'}$ and then applying operations $\boldsymbol{qs}$.

From the commutative property of the effectful operations, and the fact that the operation sequences $\boldsymbol{ps}$ and $\boldsymbol{qs}$ have been normalized, we conclude that

- The data value (3) is equivalent to (1). This is because that by exchanging the role of q'_j in (1) and p'_k (3), we can get the same data value.
- The data value (3) is equivalent to (2) provided $D_P = D_Q = D$. This is because that by moving q_j in (3) to the left before p'_i we have (2).

Hence, the replicas after independent synchronization by P and Q have the same data value. That is, our transformation assures the confluence property of the effectful set operations (Fig.1).

Recall the small example in Section 2: P and Q start with $D_P = D_Q = \{1, 2\}$, and P does $\cup\{3\}$ and then $\setminus\{3\}$, and Q does $\setminus\{2\}$ and then $\cup\{3\}$. We see that operations $\cup\{x\}$ and $\setminus\{x\}$ here are in fact effectful. So, P performs operations $\boldsymbol{ps} = \langle \oplus 3, \ominus 3 \rangle$ to have $D'_P = \{1, 2\} \oplus 3 \ominus 3 = \{1, 2\}$, and Q performs concurrentry $\boldsymbol{qs} = \langle \ominus 2, \oplus 3 \rangle$ to have $D'_Q = \{1, 2\} \ominus 2 \oplus 3 = \{1, 3\}$. As our first step to do is to normalize the operation sequences: $\boldsymbol{ps}$ becomes $\langle !, ! \rangle$ and $\boldsymbol{qs}$ remains as it is. The above synchronization procedure derives $\boldsymbol{qs'} = \boldsymbol{qs}$ since there is no elements in $\boldsymbol{ps}$ that is equal to q_j. Hence, P's local data becomes $D'_P \ominus 2 \oplus 3 = \{1, 2\} \ominus 2 \oplus 3 = \{1, 3\}$. Q produces $\boldsymbol{ps'} = \langle !, ! \rangle$ from $\boldsymbol{ps}$, and gives local data $D'_Q \; ! \; ! = \{1, 3\} \; ! \; ! = \{1, 3\}$. Thus our synchronization gives a confulence $\{1, 3\}$ as expected.



In P
In Q
D
$\odot p_1$
$\odot p_2$
$\odot q_1$
$\odot q_2$
$D \odot p_1 \odot p_2 \odot \cdots \odot p_m$
$D \odot q_1 \odot q_2 \odot \cdots \odot q_n$
$\odot q'_1$
$\odot q'_2$
$\odot p'_1$
$\odot p'_2$
$D \odot p_1 \odot p_2 \odot \cdots \odot p_m \odot q'_1 \odot q'_2 \odot \cdots \odot q'_n = D \odot q_1 \odot q_2 \odot \cdots \odot q_n \odot p'_1 \odot p'_2 \odot \cdots \odot p'_m$

Fig. 1. Confluence of Updates by Synchronization

A Digression Given lists of integers $xs = [x_1, x_2, ..., x_m]$ and $ys = [y_1, y_2, ..., y_n]$ each has no duplicate elements in itself, but with possible duplicates between xs and ys. Then, how can we calculate the sum of different integers in concatenated list $xs +\!\!+ ys$? For example, $xs = [3, 1, 4, 5, 9, 2]$ and $ys = [8, 2, 7, 6, 1]$ have no duplicates in themselves, but 1 and 2 appear in $xs +\!\!+ ys$.

We may write code[2]:

- Compute first $ys' = [y'_1, y'_2, ...y'_n]$ by replacing y_j with 0 if it appears in xs or keeping it in ys' for $j = 1, 2, \cdots$.
- And then compute $sum(xs +\!\!+ ys')$.

For the example above, $sum([3, 1, 4, 5, 9, 2] +\!\!+ [8, 0, 7, 6, 0])$ gives the answer. Note that by computing xs' from xs and ys, $sum([3, 0, 4, 5, 9, 0] +\!\!+ [8, 2, 7, 6, 1])$ gives the same result.

This algorithm helps us to understand our procedure for our conflict-free synchronization of set operations.

3.4 Eventual Consistency over Distributed Peers

Note that update synchronization in each peer does not necessarily processed at the same time, rather each peer does it when convenient. We can see that independent update synchronization eventually arrives at the same data after no more local operations are performed in both under the conditions:

- D_P and D_Q have been synchronized at least once, and
- All the local operations $\boldsymbol{ps}$ in P and $\boldsymbol{qs}$ in Q performed since the last synchronization are sent to and received from each other with the order kept and no element lost.

The above observation comes from the fact that our procedure for synchronization to compute

$$D_P \odot p_1 \odot p_2 \odot \cdots \odot p_m \odot q'_1 \odot q'_2 \odot \cdots \odot q'_n.$$

from $\boldsymbol{ps}$ and $\boldsymbol{qs}$ can be divided into segments in any ways such as

$$D_P \odot p_1 \odot q'_1 \odot p_2 \odot \cdots \odot p_m \odot q'_2 \odot \cdots \odot q'_n.$$

[2] Haskellers may solve this quiz by $sum \cdot nub$. Haskell's standard library provides function nub for removing duplicates in a list. But this does not help us here to understand our synchronization.

Managament of Local and Remote Operations To synchronize the local data, or replicas of independent peers, we have to know the shared data which are synchronized last time. These are the roots to which local operations performed and then followed by remote operationsfor synchronization.

Since P and Q concurrently run, they do not always perform updates at the same time. So, they need to keep operations since the last synchronization until the next. Then, when and how can we shorten the operation sequence?

The revision number $\#D$ of the local data D helps us to recognize the state of synchronization. It is incremented every time a local operation p_i is performed including transformed remote operations q'_j. By maintaining the pair of local and revision numbers $\langle \#D_P, \#D_Q \rangle$ of P and sending this with operations to the partner Q, we can recognize which part of local operations are no more needed (Fig.2)

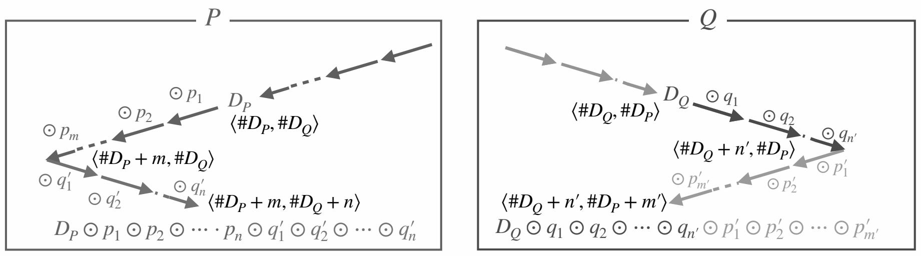

P
Q
D_P
$\langle \#D_P, \#D_Q \rangle$
$\odot p_1$
$\odot p_2$
$\odot p_m$
$\langle \#D_P + m, \#D_Q \rangle$
$\odot q'_1$
$\odot q'_2$
$\odot q'_n$
$\langle \#D_P + m, \#D_Q + n \rangle$
$D_P \odot p_1 \odot p_2 \odot \cdots \cdot p_n \odot q'_1 \odot q'_2 \odot \cdots \odot q'_n$
D_Q
$\langle \#D_Q, \#D_P \rangle$
$\odot q_1$
$\odot q_2$
$\odot q_{n'}$
$\langle \#D_Q + n', \#D_P \rangle$
$\odot p'_1$
$\odot p'_2$
$\odot p'_{m'}$
$\langle \#D_Q + n', \#D_P + m' \rangle$
$D_Q \odot q_1 \odot q_2 \odot \cdots \odot q_{n'} \odot p'_1 \odot p'_2 \odot \cdots \odot p'_{m'}$

Fig. 2. Revision Numbers with Local and Remote Operations

Synchronization of Distributed Peers So far, we have seen solely how synchronization works between P and Q.

If P has another connection to peer R, P needs to synchronize P's local data and R's local data as P has done with Q.

Consider P begins to synchronize its current local data just after the synchronization with Q. The synchronization with Q began with revision numbers $\langle \#D_P, \#D_Q \rangle$ and local data $D_P \odot \boldsymbol{ps}$ and produced the local data $D_P \odot \boldsymbol{ps} \odot \boldsymbol{qs'}$, where $\odot \boldsymbol{ps}$ represents applying operations of $\boldsymbol{ps}$.

The last synchronized data of P with R is not necessarily the same as with Q. And operation sequence $\boldsymbol{qs'}$ performed in the synchronization with Q has been sent to R as the propagation of Q's updates to other peers.

Let D'_P and D_R has been synchronized, and $\boldsymbol{ps*}$ is the operations performed in P satisfying $D'_P \odot \boldsymbol{ps*} = D_P \odot \boldsymbol{ps} \odot \boldsymbol{qs'}$.

Then, the synchronization process starts from D'_P and local operations $\boldsymbol{ps*}$ already performed on D'_P and remote operations $\boldsymbol{rs}$ sent from R. This proceeds the same as P did for Q with D_P, $\boldsymbol{ps}$ and $\boldsymbol{qs}$. This time, P produces $D'_P \odot \boldsymbol{ps*} \odot \boldsymbol{rs'}$ with new operations $\boldsymbol{rs'}$ from $\boldsymbol{rs}$ as shown in Fig.3. The operations $\boldsymbol{rs'}$ are also propagated to other peers including Q.

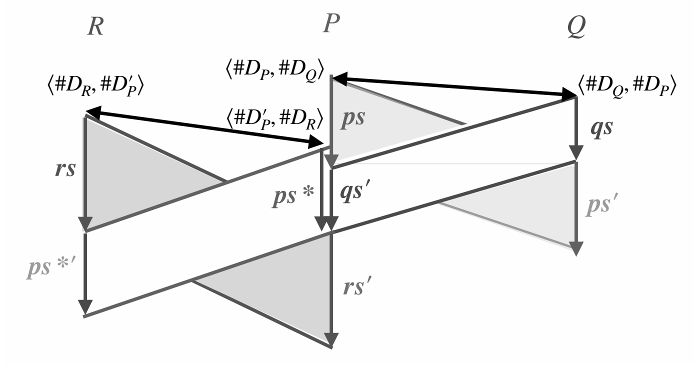

R
P
Q
$\langle \#D_R, \#D'_P \rangle$
$\langle \#D_P, \#D_Q \rangle$
$\langle \#D'_P, \#D_R \rangle$
$\langle \#D_Q, \#D_P \rangle$
ps
qs
rs
$ps*$
qs'
ps'
$ps*'$
rs'

Fig. 3. Synchronization of Distributed Peers

Thus, we conclude that our synchronization scheme works over distributed peers, and this leads to the eventual consistency of the whole system.

4 Remarks

We can enjoy *Conflict-free Collaborative Set Sharing* (CCSS) simply by

– When inserting data, first check whether it is not there yet, and
– When deleting data, first check whether it really is there already.

When we use these operations for updates on the local data, failure of the check tells us that the operation is *invalid* and should not be done, or rather it tells us that it is not necessary for our intended updates.

It is very simple to do whatever data with ubiquitous set semantics. As a matter of course, the oft-used SQL table is the case.

We can easily extend our CCSS to place transformations at the gateway of the peer for sending and receiving operations for controlling shared data. Data sharing with the *Dejima* in BISCUITS Project is an example [8].

Also we can extend our CCSS to include mechanisms for choosing one from grouped data according to preferences, e.g., when or who inserts the data. We may call such a strategy *Semantic Resolution*. For example, the LWW Set (Last-Write-Win) CRDT can be realized by attaching the logical time stamp as meta-data to the data value with the key for grouping data. A process is provided for choosing one from candidates inserted by $\oplus(v,k,t)$, $\oplus(v',k,t')$, $\cdots$. We can choose (v,k,t) if $t > t'$ by performing local operation $\ominus(v',k,t')$ for LWW.

Implementation of CCSS peers would be straightforward and an exercise of standard network programming.

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